Norwegian University of Science and Technology
Departement of Mathematical Sciences

TMA4110
Calculus 3
Autumn 2010

**Exercise set 10 – Week 44**

## Edwards & Penney, section 4.4

**7,17,19,23,30**

## Edwards & Penney, section 5.1

**3,19,23,30**

## Exam problems

Dec. 03, problem 5 Given the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix},\ \mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix},\ \mathbf{v}_3 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix},\ \mathbf{v}_4 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix},\ \mathbf{v}_5 = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} \quad \text{og} \quad \mathbf{b} = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}.$$

**a)** Find a basis for the subspace $V \subseteq \mathbb{R}^3$ spanned by the vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_5$, and also a basis for the subspace $V^\perp$ (the orthogonal complement of $V$ in $\mathbb{R}^3$).

**b)** Find the orthogonal projection of $\mathbf{b}$ into the subspace $V$.

**c)** Let $V \subseteq \mathbb{R}^n$, and assume we're given vectors $\mathbf{v} \in V$ and $\mathbf{w} \in V^\perp$. Show that if neither $\mathbf{v}$ nor $\mathbf{w}$ is the zero vector, then $\mathbf{v}$ and $\mathbf{w}$ are linearly independent.

May 01, problem 4 Given the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 5 \\ 1 & 5 & -9 & -4 \\ 2 & 5 & 2 & 7 \end{bmatrix}.$$

**a)** Find Null($A$). What is the solution set of

$$A\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}?$$

**b)** Find a basis for Col($A$), Row($A$) and Row($A$)$^\perp$).

## Multiple-choice questions

1 For which value(s) of $k$ are the vectors $\mathbf{u} = (2, 2, -1, k)$ and $\mathbf{v} = (k, 1, 1, k)$ mutually orthogonal?

**A:** $k = 1$ **B:** $k = \pm 1$ **C:** $k = -1$ **D:** no $k$