(1) Consider the equation

$$\frac{\partial^2 u}{\partial t^2} - c^2 \Delta u = F(\bar{x}, t) \quad \text{in } \mathbb{R}^3 \times [0, \infty).$$

Suppose that u_1 and u_2 are two solutions with initial values

$$u_j(\bar{x}, 0) = g_j(\bar{x}), \quad \frac{\partial u_j(\bar{x}, 0)}{\partial t} = h_j(\bar{x}), \quad j = 1, 2,$$

when $\bar{x} \in \mathbb{R}^3$. Use Kirchhoff's formula

$$\bar{u}(\bar{x}, t) = \frac{\partial}{\partial t}\left\{ \frac{t}{4\pi c^2 t^2} \oint\!\!\oint_{\partial B(\bar{x}, ct)} g \, dS_{ct} \right\} + \frac{t}{4\pi c^2 t^2} \oint\!\!\oint_{\partial B(\bar{x}, ct)} h \, dS_{ct}$$

for the difference $u_2 - u_1$ to prove the STABILITY estimate

$$|u_2(\bar{x}, t) - u_1(\bar{x}, t)| \leq t \max_{\bar{x}} |h_2(\bar{x}) - h_1(\bar{x})|$$

$$+ \max_{\bar{x}} |g_2(\bar{x}) - g_1(\bar{x})| + ct \max_{\bar{x}} |\nabla (g_2(\bar{x}) - g_1(\bar{x}))|$$

---

(2) Let $\Omega \subset \mathbb{R}^n$ be a smooth bounded domain and consider the problem

$$\begin{cases} u_{tt} - c^2 \Delta u + \alpha u_t = 0 & \text{in } \Omega \times (0, \infty) \\ u(\bar{x}, 0) = 0, \quad u_t(\bar{x}, 0) = 0 & \text{when } x \in \Omega \\ u(x, t) = 0 & \text{on } \partial\Omega \times (0, \infty) \end{cases}$$

Show that the solution $u \equiv 0$ if $\alpha \geq 0$, provided that $u \in C^2(\bar{\Omega} \times [0, \infty))$. <u>Hint</u>:

$$\mathcal{E}(t) = \frac{1}{2} \int_{\Omega} \left(u_t(x,t)^2 + c^2 |\nabla u(x,t)|^2 \right) d\bar{x}$$

<u>Remark</u>: $\alpha u_t(\bar{x}, t)$ represents dissipation.