Assume that $u \geq 0$ and $\Delta u = 0$ in Ω. Let $B(0,R) \subset\subset \Omega$. Show that Poisson's formula

$$u(x) = \frac{R^2 - |x|^2}{\alpha_n R} \oint_{|\xi|=R} \frac{u(\xi)\, dS_R(\xi)}{|x-\xi|^n},$$

where $\alpha_n = \text{area}(\partial B(0,1))$, implies Harnack's inequality
(area of the unit sphere in $\mathbb{R}^n$)

$$\frac{R^{n-2}(R-|x|)}{(R+|x|)^{n-1}} u(0) \leq u(x) \leq \frac{R^{n-2}(R+|x|)}{(R-|x|)^{n-1}} u(0).$$

Furthermore,

$$u(x) \leq \left(\frac{R+r}{R-r}\right)^n u(y), \quad |x| \leq r, \ |y| \leq r,$$

where $0 \leq r < R$. Deduce

THM (Liouville) If $u \geq 0$ and $\Delta u = 0$ in the whole $\mathbb{R}^n$, then u reduces to a constant.