TMA4145 Linear Methods
Fall 2020

Exercise set 1

Norwegian University of Science and Technology
Department of Mathematical Sciences

Please justify your answers! The most important part is *how* you arrive at an answer, not the answer itself.

1 Let X, Y and Z be sets.

a) Show that $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$.

b) Show that $X \backslash (Y \cup Z) = (X \backslash Y) \cap (X \backslash Z)$.

2 Let $f : X \to Y$ be a function, let B be a subset of Y, and let $\{B_i\}_{i \in I}$ be a family of subsets of Y.

a) Prove that

$$f^{-1}\left(\bigcap_{i \in I} B_i\right) = \bigcap_{i \in I} f^{-1}(B_i).$$

b) Prove that $f(f^{-1}(B)) \subseteq B$, and if f is surjective then equality holds. Show by example that equality need not hold if f is not surjective.

3 Let $f : X \to Y$ be a function, let A be a subset of X, and let $\{A_i\}_{i \in I}$ be a family of subsets of X.

a) Prove that

$$f\left(\bigcup_{i \in I} A_i\right) = \bigcup_{i \in I} f(A_i).$$

b) Prove that $A \subseteq f^{-1}(f(A))$, and if f is injective then equality holds. Show by example that equality need not hold if f is not injective.

4 Show that the sets $\mathbb{Z}$ of integers and $\mathbb{Q}$ of rational numbers are countable.

5 Show that the Cartesian product of two (infinite) countable sets is countable.

6 **(Challenge)** Prove that the closed interval $[0, 1]$ and the open interval $(0, 1)$ have the same cardinality by finding a bijection $f : [0, 1] \to (0, 1)$. *Hint: Do not try to create a continuous function f.*