# Eigenvalues and Eigenvectors

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# Instructions

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**EIGENVALUES & EIGENVECTORS**

Let $V$ be a vector space and let $\mathbf{u}, \mathbf{v}, \mathbf{w}$, and $\mathbf{x}$ be vectors in $V$. Let $A = \begin{pmatrix} 1 & 2 \\ 0 & -3 \end{pmatrix}$,

$B = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}$, $\mathbf{u} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$, $\mathbf{w} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, and $\mathbf{x} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$. Then

$$A\mathbf{u} = \begin{pmatrix} 3 \\ -6 \end{pmatrix} = -3 \begin{pmatrix} -1 \\ 2 \end{pmatrix} = -3\mathbf{u}$$

$$A\mathbf{v} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \neq \lambda \mathbf{v}, \quad \text{for any scalar } \lambda$$

$$B\mathbf{w} = \begin{pmatrix} 3 \\ 6 \end{pmatrix} = 3 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = 3\mathbf{w}$$

$$B\mathbf{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} = 0 \begin{pmatrix} 1 \\ -1 \end{pmatrix} = 0\mathbf{x}$$

- The vector $\mathbf{u}$ is an eigenvector of the matrix $A$ and -3 is its corresponding eigenvalue.

- The vector $\mathbf{w}$ is an eigenvector of the matrix $B$ and 3 is its corresponding eigenvalue.

- The vector $\mathbf{x}$ is an eigenvector of the matrix $B$ and 0 is its corresponding eigenvalue.

- The vector $\mathbf{v}$ is **NOT** an eigenvector of the matrix $A$.

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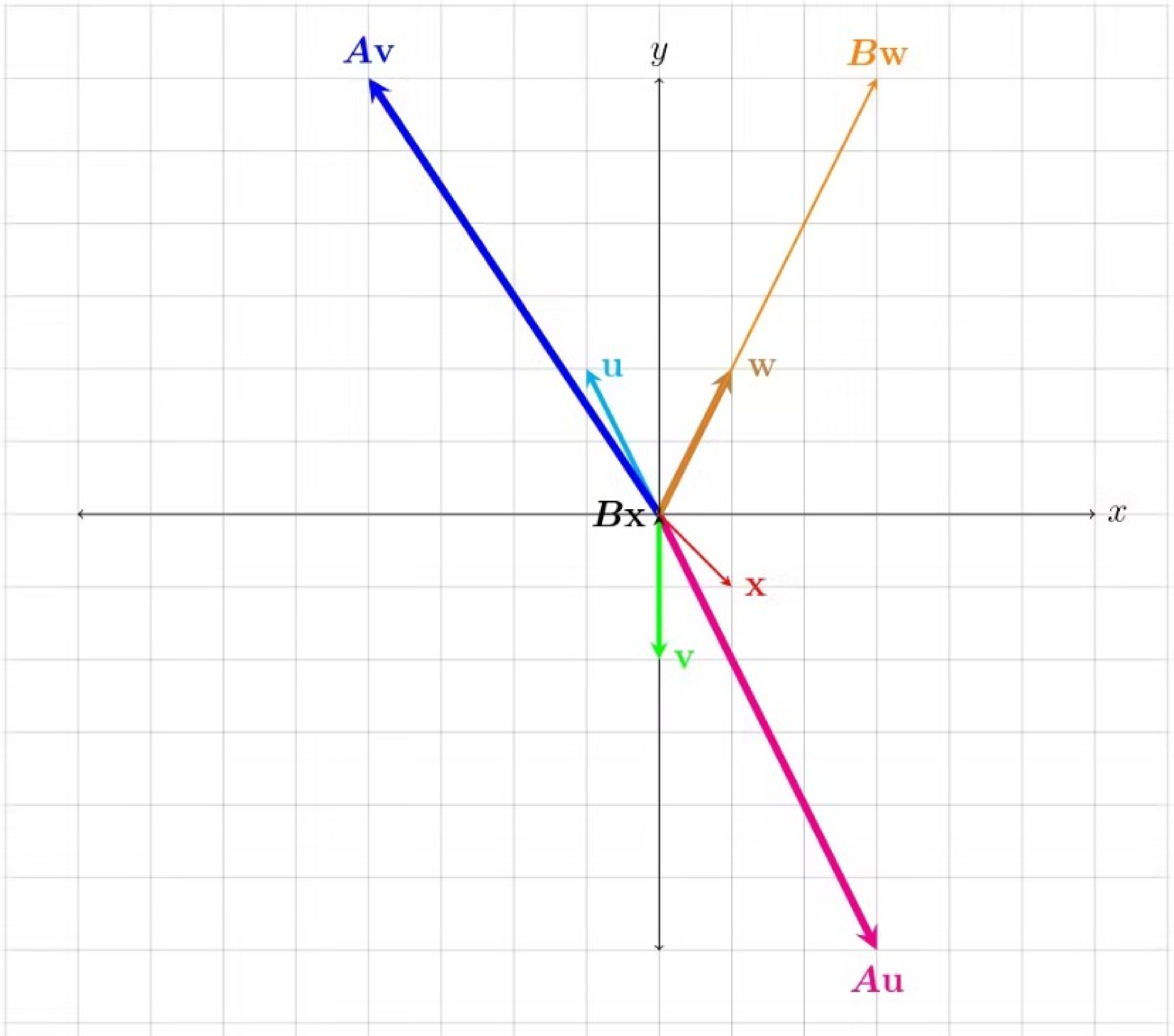
$$\mathbf{u} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \longleftrightarrow A\mathbf{u} = \begin{pmatrix} 3 \\ -6 \end{pmatrix} = -3\mathbf{u}; \quad \mathbf{v} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \longleftrightarrow A\mathbf{v} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$$

$$\mathbf{w} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \longleftrightarrow B\mathbf{w} = \begin{pmatrix} 3 \\ 6 \end{pmatrix} = 3\mathbf{w}; \quad \mathbf{x} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \longleftrightarrow B\mathbf{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} = 0\mathbf{x}$$



# Eigenvalues and Eigenvectors

1. An eigenvector of an $n \times n$ matrix A is a nonzero vector $\mathbf{v}$: $A\mathbf{v} = \lambda\mathbf{v}$.
2. A scalar $\lambda$ is called an eigenvalue of A if $\exists$ a nontrivial solution $\mathbf{v}$ of $(A - \lambda I)\mathbf{v} = 0$.
3. $\mathbf{v}$ is called an eigenvector corresponding to $\lambda$.

# If $(A - \lambda I)\mathbf{v} = 0$ has a unique solution, then $\lambda$ is an eigenvalue of $A$.

TRUE

FALSE

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# If $(A - \lambda I)\mathbf{v} = 0$ has infinitely many solutions, then there exists at least one eigenvector $\mathbf{v}$ of $A$.

TRUE

FALSE

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# If the matrix $(A - \lambda I)$ is invertible, then we can find _ _ _ _ _ eigenvalue(s) for $A$.

If $\det(A - \lambda I) = 0$, then we can find eigenvalues $\lambda$ for $A$.

TRUE

FALSE

# If the matrices A and B are row equivalent, then they have the same eigenvalues.

TRUE ✗ FALSE ✓

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# Eigenvalues and Eigenvectors

4. Algebraic multiplicity of $\lambda$: Is the number of times $\lambda$ appears as a root of $\det(A - \lambda I) = 0$

5. Geometric multiplicity of $\lambda$: Is the dimension of $Nul(A - \lambda I)$

6. Eigenspace of $A$ for $\lambda$ : Is the space $Nul(A - \lambda I)$. If $T : V \to V$ is a linear transformation, then the eigenspace is $\{\mathbf{v} \in V | T(\mathbf{v}) = \lambda \mathbf{v}\}$

7. $1 \leq \lambda_{G.M} \leq \lambda_{A.M}$

# What is the geometric multiplicity of the eigenvalue 3 of the matrix $\begin{pmatrix} 3 & 2 \\ 0 & 3 \end{pmatrix}$?

The correct answer is: 1

# What is the geometric multiplicity of the eigenvalue 3 the matrix $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$?

The correct answer is: 2

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# If $\lambda$ is an eigenvalue with corresponding eigenvector $\mathbf{v}$, then $Sp\{\mathbf{v}\}$ is a set of all eigenvectors corresponding to $\lambda$.

✗ TRUE

✓ FALSE

# If 0 is an eigenvalue, then the matrix $A$ is invertible.

TRUE

FALSE

# What are the eigenvalues of

$$A = \begin{pmatrix} 2 & 0 & 3 \\ 0 & -1 & 2 \\ 0 & 0 & 0 \end{pmatrix}?$$

- ✗ −2, 1, 0
- ✓ 2, −1, 0
- ✗ −2, 1
- ✗ 2, 1

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# Eigenvalues & Eigenvectors

8. **Theorem :** Eigenvalues of a triangular matrix $A$ are the entries on the leading diagonal of $A$.

9. **Theorem :** The eigenvalues of a diagonal matrix are the diagonal entries.

10. **Theorem :** If $\mathbf{v}_1, \ldots, \mathbf{v}_k$ are eigenvectors that correspond to distinct eigenvalues $\lambda_1, \ldots, \lambda_k$ of an $n \times n$ matrix $A$, then the set $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is linearly independent

# Eigenvalues & Eigenvectors

11. **Theorem :** A complex $n \times n$ matrix $A$ always has $n$ eigenvalues (counted with Algebraic Multiplicity)

12. **Theorem :** The complex eigenvalues of a real matrix comes in complex conjugate form

**If $A$ is a $5 \times 5$ matrix and $rank\, A = 2$, then 0 is an eigenvalue of $A$.**

TRUE

FALSE

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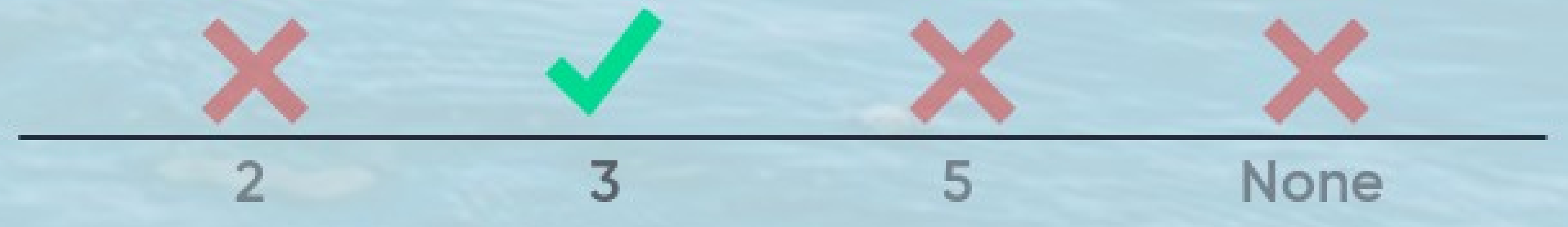
# Let $A$ be a $5 \times 5$ matrix and let $rank\, A = 2.$ Then the geometric multiplicity of 0 is _ _ _ _



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# Let $A$ be a $5 \times 5$ matrix and let $rank\, A = 2$. If there are 3 distinct eigenvalues of $A$, then their algebraic multiplicities can be listed as

| ✓ | ✗ | ✗ |
|---|---|---|
| 1, 1, and 3 | 1, 1, and 2 | 0, 2, and 3 |

# CONGRATULATIONS!

## No results yet

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1. Let $\mathbf{v} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$ be an eigenvector corresponding to the eigenvalue 0 of the matrix $A = \begin{pmatrix} 2a-b & 6 & 0 \\ -1 & -1 & 1 \\ 2 & b-a & -8 \end{pmatrix}$. Find $a$ and $b$.

2. i. Let $A$ be an invertible matrix and let $\lambda$ be an eigenvalue of $A$. Prove that $\frac{1}{\lambda}$ is an eigenvalue of $A^{-1}$, the inverse of $A$.

   Let $k \geq 1$ be an integer.

   ii. If 0 is an eigenvalue of the matrix $A$, then 0 is an eigenvalue of the matrix $A^k$.

3. Let the matrix $A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 3 \\ 0 & 4 & 0 \end{pmatrix}$.

   i. Find the eigenvalues of $A$.

   ii. Find the eigenspaces corresponding to the eigenvalues of $A$.

# QUESTIONS - 35 MINUTES