Diagonalization

NTNU
Kunnskap for en bedre verden

Instructions

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Question – 5 minutes

1. Let A be a real or complex square matrix and let $k \geq 1$ be an integer.

i. Prove that if λ is an eigenvalue of the matrix A, then λ^k is the eigenvalue of the matrix A^k.

ii. Prove that if A^k is the zero matrix, then 0 is the only eigenvalue of A.

Diagonalization

1. **Similar matrices:** Let A and B be square matrices. A is similar to B if there is an invertible matrix P:

$$P^{-1}AP = B \text{ or } A = PBP^{-1}.$$

2. **Theorem :** If A and B are similar matrices, then they have the same characteristic polynomial and hence the same eigenvalues.

3. A square matrix is diagonalizable if $A = PDP^{-1}$, where P is some invertible matrix and D is some diagonal matrix.

If the matrices A and B have the same eigenvalues, then they are similar.

TRUE

FALSE

If the matrices A and B are row equivalent, then they are similar.

TRUE

FALSE

Leaderboard

No results yet

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Diagonalization

4. An $n \times n$ matrix A is diagonalizable if and only if:

   i. A has n linearly independent eigenvectors.

   ii. There exist an eigenvector basis of $\mathbb{R}^n$.

   iii. The Algebraic multiplicity of every eigenvalue equals its corresponding Geometric multiplicity.

5. $\textbf{Theorem}$: An $n \times n$ matrix with n distinct eigenvalues is diagonalizable.

If A is a diagonal matrix, then A is invertible.

TRUE

FALSE

If A is the zero matrix, then A is diagonalizable.

TRUE

FALSE

If A is a diagonal matrix, then A is diagonalizable.

TRUE

FALSE

Leaderboard

No results yet

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The matrix $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & 2 \\ 0 & 0 & 4 \end{pmatrix}$ is diagonalizable.

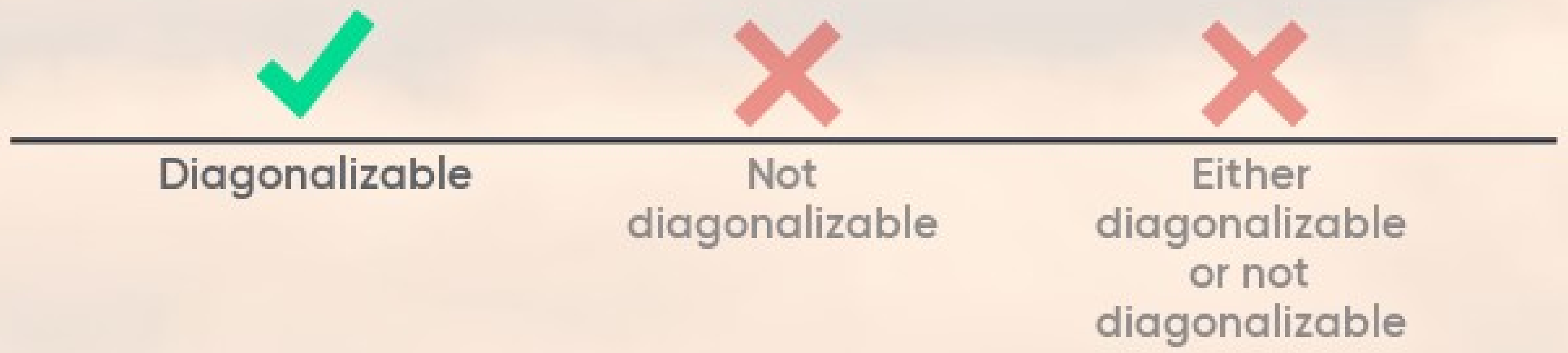
TRUE

FALSE

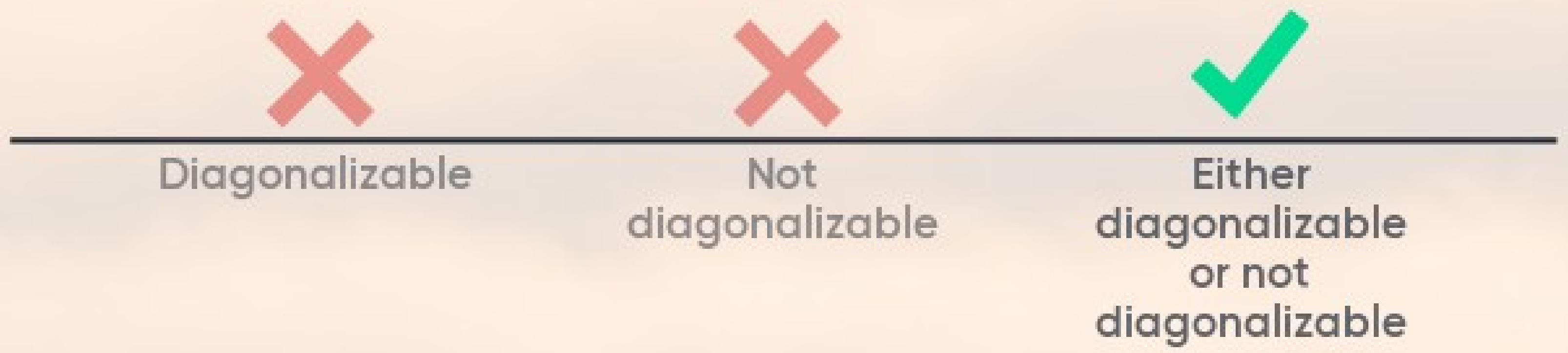
The matrix $A = \begin{pmatrix} 2 & 0 & 0 \\ 1 & -3 & 0 \\ 2 & 1 & 0 \end{pmatrix}$ is diagonalizable.

TRUE FALSE

Let A be a 5×5 matrix with three distinct eigenvalues. If two of the eigenspaces are 2-dimensional each, then A is _ _ _ _ _



Let A be a 5×5 matrix with two distinct eigenvalues. If one of the eigenspaces has dimension 1, then A is _ _ _ _ _



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Diagonalization

6. **Symmetric Matrix :** Is a square matrix A such that $A = A^T$.

7. **Hermitian Matrix :** $A = \bar{A}^T$

8. **Theorem :** If A is symmetric, then any two eigenvectors from different eigenspaces are orthogonal.

9. **Orthogonally Diagonalizable :** An $n \times n$ matrix A is orthogonally diagonalizable if there exist a diagonal matrix D and an orthogonal matrix P satisfying $P^T = P^{-1}$ such that

$$A = PDP^T$$

Diagonalization

10. **Theorem :** An $n \times n$ matrix A is orthogonally diagonalizable if and only if A is symmetric.

11. **Theorem :** Let A be a real symmetric matrix. Then $A = PDP^T$, where P is an orthogonal matrix.

12. **Theorem :** An $m \times n$ matrix A has orthonormal columns if and only if $A^T A = I$.

13. **Orthogonal** or **Orthonormal matrix :** Is a real square matrix A with orthonormal rows and columns.
i.e. $A^T = A^{-1}$.

The matrix $\begin{pmatrix} 1 & i & 2 \\ i & 0 & 1 \\ 2 & 1 & 3 \end{pmatrix}$ **is Hermitian.**

TRUE

FALSE

CONGRATULATIONS!

No results yet

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QUESTIONS - 45 MINUTES

1. Let A be a real or complex square matrix and let $k \geq 1$ be an integer.

i. Prove that if λ is an eigenvalue of the matrix A, then λ^k is the eigenvalue of the matrix A^k.

ii. Prove that if A^k is the zero matrix, then 0 is the only eigenvalue of A.

2. Let $\mathbb{P}_2 = \{\mathbf{p}(x) = a_0 + a_1 x + a_2 x^2 \mid a_0, a_1, a_2 \in \mathbb{R}\}$ be the vector space consisting of all polynomials of degree at most 2. Let $T : \mathbb{P}_2 \to \mathbb{P}_2$ be a linear transformation defined by

$$T(\mathbf{p}(x)) = (1+4x)\mathbf{p}(x) - x(1+3x)\mathbf{p}'(x) + \left(\frac{1}{2} + x(1+x^2)\right)\mathbf{p}''(x).$$

i. Verify that $T(\mathbf{p}(x)) \in \mathbb{P}_2$.

ii. Find A, the matrix representation of T relative to the basis $\{1, x, x^2\}$.

iii. Find the eigenvalues of A.

iv. Find the eigenspaces corresponding to each distinct eigenvalue.

v. Is the matrix A diagonalizable? Justify your answer.

3. Let $B = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$.

i. Find the eigenvalues of B.

ii. Find the eigenspaces corresponding to each distinct eigenvalue.

iii. Is the matrix B diagonalizable? Justify your answer.